RELIABILITY BASED SAFETY FACTOR AGAINST SOIL LIQUEFACTION

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INTRODUCTION

Simplified procedures, originally proposed by Seed and Idriss (1971), updated by Seed et al. (1985) and Youd et al. (2001) using the standard penetration test (SPT), are frequently used to evaluate the liquefaction potential of soils. With a deterministic method, liquefaction of soil is predicted to occur if the factor of safety (FS), which is the ratio of the cyclic resistance ratio (CRR) over the cyclic stress ratio (CSR), is less than or equal to one. No soil liquefaction is predicted if $FS > 1$. Despite the significant uncertainties in the different variables involved in this deterministic method, practical liquefaction risk assessment is still rooted in deterministic analysis. Reliability calculations provide a means of evaluating the combined effects of uncertainties and provide a logical framework for choosing factors of safety that are appropriate for the degree of uncertainty and the consequences of failure. Thus, as an alternative or a supplement to the deterministic assessment, a probabilistic assessment of liquefaction potential may be performed in which the liquefaction potential is assessed in terms of the probability of liquefaction. The results of such a probabilistic assessment of liquefaction potential could lead to better engineering decisions.

The most widely used method of reliability analysis for soil liquefaction is the first order second moment (FOSM) method, Baecher and Christian (2003). Using FOSM, the variability in CSR and CRR can be easily assessed and reliability analysis can be performed thereafter to calculate the probability of liquefaction. The objectives of the present study is to apply such approach to compute probability of liquefaction and to present a modified factor of safety based on reliability approach, that considers variability of CSR and CRR for a specified level of risk .

DETERMINISTIC APPROACH FOR SOIL LIQUEFACTION

In the liquefaction evaluation, the cyclic stress ratio CSR has been proposed by Seed and Idriss (1971) as

$$CSR = 0.65 \frac{\sigma_v}{\sigma_v^{'}} \frac{a_{\max}}{g} \gamma_d \tag{1}$$

where σ_v =total vertical stress; $\sigma_v^{'}$ =effective vertical stress; $a_{\max}$ = peak horizontal ground surface acceleration; g= acceleration of gravity; and γ_d=nonlinear shear stress mass participation factor (or stress reduction factor).

Cyclic resistance ratio (CRR), the capacity of soil to resist liquefaction, can be obtained from corrected blow count, $(N_1)_{60}$ from empirical correlations, proposed by Seed et al. (1985). CRR curves are proposed for granular soils with the fines content of 5% or less, 15%, and 35% and for magnitude 7.5 earthquakes. The CRR curves for fines content < 5% (clean sands) can be approximated by, Youd et al. (2001) as

$$CRR_{7.5} = \frac{1}{34-(N_1)_{60}} + \frac{(N_1)_{60}}{135} + \frac{50}{\left[10.(N_1)_{60}+45\right]^2} - \frac{1}{200} \tag{2}$$

for $(N_1)_{60}$ <30. For $(N_1)_{60} \geq 30$, clean granular soils are classified as non-liquefiable. The deterministic factor of safety (FS_d) against liquefaction in terms of CSR and CRR is defined by

$$FS_d = \frac{CRR_{7.5}}{CSRN}.K_\sigma.K_\alpha \tag{3}$$

where CSRN is the normalized CSR for earthquakes of magnitude 7.5 (CSR/MSF); MSF is the magnitude scaling factor; K_σ is the correction factor for effective overburden; and K_α is the correction factor for sloping ground. An example is presented here using deterministic and probabilistic approaches. Typical bore log data from a site located at the Nepal television studio building, Singhdurbar, Kathmandu, is shown in Fig. 1. A liquefiable sandy layer exists from a depth of 4m to 14m. The water table is at a depth of 3.25m. The site has been analyzed for $a_{\max}$ = 0.2g, and $M_w = 7$. The soil parameters and the factors of safety against liquefaction using a deterministic method (FS_d) are shown in Fig. 1. The reliability analysis is then performed using FOSM method. Table 1 shows the mean and coefficient of variation (the ratio of standard deviation over mean value) of the parameters considered in the analysis.

RELIABILITY APPROACH FOR SOIL LIQUEFACTION

The first order second moment (FOSM) method is a relatively simple method for including the effects of variability of input variables on a resulting dependent variable. Using this FOSM method, the mean and COV of CSR are given by

$$\mu_{CSRN} = 0.65\frac{\mu_{a_{\max}}}{g}\frac{\mu_{\sigma_v'}}{\mu_{\sigma_v}}\frac{\mu_{\gamma_d}}{\mu_{MSF}} \tag{4}$$

$$V_{CSRN}^2 = V_{a_{\max}}^2 + V_{\sigma_v'}^2 + V_{\sigma_v}^2 + V_{\gamma_d}^2 + V_{MSF}^2 - 2\rho_{\sigma_v\sigma_v'}V_{\sigma_v'}V_{\sigma_v} \tag{5}$$

where μ and V represent the corresponding mean and coefficient of variation (ratio of mean and standard deviation), respectively, and $\rho_{\sigma_v\sigma_v'}$ represents the correlation coefficient between total and effective stress. Since σ_v , σ_v' are directly computed from bore log and laboratory test data, they should be regarded as deterministic values with no variance. The uncertainty in the CSR is mainly governed by the uncertainty involved in estimating the peak ground acceleration, $a_{\max}$, for a given earthquake of magnitude M, magnitude scaling factor (MSF), and shear mass participation factor (γ_d).

Similarly, the resistance of soil against liquefaction depends on a representative SPT value $(N_1)_{60}$. The mean value of CRR can be calculated by using Eq. (3) - the most popular method –with the mean value of $(N_1)_{60}$. However, it should be noted that there are a large number of uncertainties involved in the performance of SPT. ASTM D1586-99 suggests that for the same apparatus, driller, and soil, the SPT blow-count can be reproduced with a coefficient of variation. There is a minimum inherent test error induced even when the specified standards are carefully observed. The range of total uncertainty in N-value that results from equipment, procedure and random measurement error may vary from 15% - 45%, Phoon and Kulhway (1999). When these errors are combined with other uncertainties arising from different coefficients used to normalize the SPT value (uncertainty in overburden correction factor, energy correction factor, borehole diameter correction factor, rod length correction factor, sample correction factor), the total uncertainties in $(N_1)_{60}$ become much higher. The mean value of CRR is calculated using mean value of $(N_1)_{60}$ in Eq. (3) and COV (V) can be calculated as $V_{CRR} = \Delta CRR/2\mu_{CRR}$, where ΔCRR is the incremental CRR [$CRR(\mu_{(N_1)_{60}} + \sigma_{(N_1)_{60}}) - CRR(\mu_{(N_1)_{60}} - \sigma_{(N_1)_{60}})$]. After obtaining the mean and COV of CSR and CRR, the lognormal reliability index can be determined, Hwang et al. (2004).

$$\beta_{LN} = \frac{\mu_{\ln Z}}{\sigma_{\ln Z}} = \frac{\mu_{\ln CRR} - \mu_{\ln CSR}}{\sqrt{\sigma_{\ln CRR}^2 + \sigma_{\ln CSR}^2}} = \frac{\ln\left[\frac{\mu_{CRR}}{\mu_{CSR}}\sqrt{\frac{V_{CSR}^2+1}{V_{CRR}^2+1}}\right]}{\sqrt{\ln[(V_{CSR}^2+1)(V_{CRR}^2+1)]}} = \frac{\ln\left[FS\sqrt{\frac{V_{CSR}^2+1}{V_{CRR}^2+1}}\right]}{\sqrt{\ln[(V_{CSR}^2+1)(V_{CRR}^2+1)]}} \tag{6}$$

where μ_{CRR} , μ_{CSR} and σ_{CRR} , σ_{CSR} are the mean values and standard deviations of CRR and CSR, respectively. The risk for liquefaction in terms of liquefaction probability P_L can be obtained from reliability index by, $P_L = 1 - \Phi(\beta)$, where $\Phi(.)$ is the cumulative normal probability (the area under the standard normal distribution curve).

Table 1: Mean and COV of input parameters for reliability analysis

	a_{max}/g	σ_v	σ'_v	γ_d	MSF	$(N_1)_{60cs}$	K_σ
Mean	0.2	f(z)	f(z)	f(z)	1.22	f(z)	f(σ'$_v$)
COV	0.2	0	0	0.1	0.05	0.25	0

RESULTS AND DISCUSSIONS

Factor of safety using deterministic method (FS_d) and the probability of liquefaction using reliability approach (FS_R) are shown in Fig. 1. It should be noted that safety factor using FOSM is the same as deterministic safety factor as FOSM only considers the mean value of CSR and CRR to obtain the mean value of safety factor and does not consider the variability of CSR and CRR to calculate the mean value of FS . However, the variability in CSR and CRR is used to calculate the variability in FS (in terms of reliability index) and corresponding probability of liquefaction. From Fig. 1, it can be seen that using deterministic safety factor method, four depths are safe against liquefaction where $FS > 1$. Similarly when it is analyzed based on probability of liquefaction, it can be seen that for $FS > 1$, liquefaction probability varies from 9% to 49%. If we assume a certain liquefaction probability, say 15%, only one depth is safe against liquefaction.

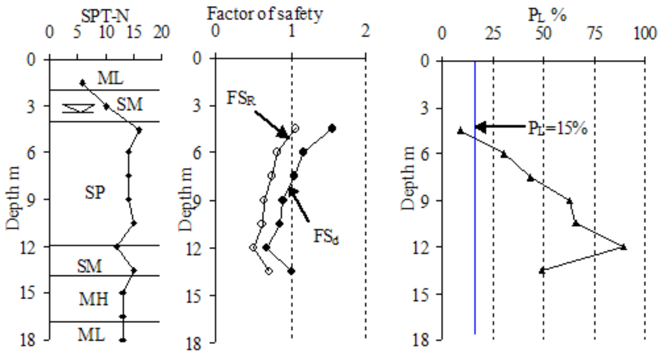

Fig. 1 Probabilistic and deterministic liquefaction analysis results

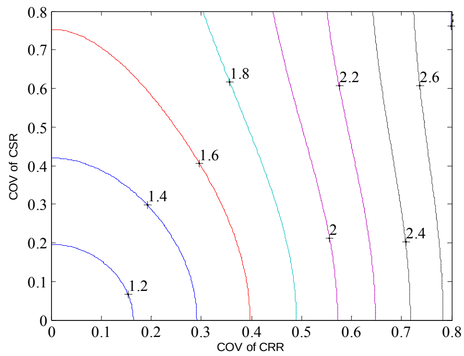

Fig. 2 Contours of the minimum safety factor required for varying COVs of CSR and CRR (P_L=15%)

In reliability analysis, the probability of liquefaction varies depending on the variability of input and soil resistance parameters. A higher factor of safety is required to obtain the same probability (or reliability) of liquefaction if the soil strength parameters and /or shear stress parameters are more variable. The liquefaction probability P_L corresponding to any given safety factor FS can be obtained from Eq. (6). For a specified risk (in terms of reliability) or for a given minimum reliability index β_{min}, the minimum safety factor ($FS_{\min}$) to be adopted for design depends on COV of CSR and CRR and can be obtained from Eq. (6) as

$$FS_{\min} = \exp\left[\beta_{\min}\sqrt{\ln[(V_{CSR}^2+1)(V_{CRR}^2+1)]} + \ln\sqrt{\frac{1+V_{CRR}^2}{1+V_{CSR}^2}}\right] \quad (7)$$

From Eq. (7), it is clear that in deterministic analyses where no variability in CSR and CRR is considered, the minimum safety factor required is equal to one. If there is variability in CSR and CRR, the safety factor required increases. As variability increases, the required safety factor also increases to achieve the same level of performance or acceptable risk in terms of probability of liquefaction. The minimum safety factor required also increases if the acceptable risk decreases. Eq. (7) is plotted in Fig. 2 and it can be seen that the minimum FS required is more dependent on variability in CRR than in CSR. For $V_{CRR} > 0.4$, the FS is dominated by V_{CRR} irrespective of the value of V_{CSR}.

Computed or designed safety factors decrease due to the variability of CSR and CRR. For example, if a safety factor of two is used in deterministic analysis, this safety factor reduces to a value of less than two depending on the variability of CSR, CRR and acceptable risk. Also, depending on the minimum safety factor required for a specified risk and the actual deterministic safety factor used without considering variability in CSR and CRR, a design safety factor based on a reliability approach (FS_R) is defined here as

$$FS_R = \frac{FS_d}{FS_{\min}} \tag{8}$$

Thus, FS_R should be greater than one to be certain of no liquefaction for a specified risk level. For example if the safety factor calculated without considering any uncertainty (FS_d) is 1.5, and the $FS_{\min}$ required when considering variability in CSR and CRR for a specified risk level is 1.6 (e.g., from Fig. 2), FS_R will be less than one and it will be regarded as unsafe. If FS_d is two, then FS_R will be 2/1.6 = 1.2 and thus regarded as safe against liquefaction. In this case, even though deterministic safety factor is 2, the actual safety factor considering reliability is only 1.2. FS_R for the example as explained in this paper is shown in Fig. 1 where the deterministic safety factor FS_d is also shown. It can be seen that when no variability in CSR and CRR is considered, four depths are safe against liquefaction (FS >1). If variability in CSR and CRR considered, only one depth is safe against liquefaction for a specified probability of liquefaction of 15% which can be represented by using FS_R instead of FS_d. For a constant FS_d, FS_R depends on $FS_{\min}$, and thus FS_R depends on specified risk or specified liquefaction probability and variability of CSR and CRR and provides a better explanation of the safety factor compared to traditional deterministic safety factors. It is thus recommended for routine liquefaction design problems.

CONCLUSIONS

A reliability analysis of soil liquefaction based on a standard penetration test N-value has been performed. A lower deterministic safety factor (say 1.5) may have a lower probability of liquefaction than a higher deterministic safety factor (say 1.6) depending on the COVs of CSR and CRR, a result supporting the notion that the deterministic safety factor is not a consistent measure of risk. A modified safety factor is recommended to be used for routine analysis that considers the COVs of CSR and CRR for a specified risk level. By using a modified safety factor, a higher value of safety factor always corresponds to higher reliability and lower probability of liquefaction. A modified safety factor greater than one always corresponds to no-liquefaction for a specified risk level which is not generally the case for deterministic safety factors if the variability in input parameters is considered.

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